Sliding Mode Control for a Quadrotor

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Abstract— **This paper presents the complete model of a minihelicopter quadrotor and describes its behavior control for the altitude by using sliding mode control. The control technique is robust against disturbances and attenuates the parametric uncertainties. The plant is a nonlinear one with state variables strongly coupled. The simulations show that the technique successfully drives the system and the altitude is well controlled as well as the rotational movements**.

Keywords- Sliding Mode Control, Control theory, UAV's , Non linear control.

I. INTRODUCTION (HEADING 1)

Quadrotors are unmanned aerial vehicle (UAV) which is widely used in several applications. Therefore, it is important to have control on them. Due to these devices are non-linear and very unstable, we have to use control techniques that must be robust against disturbances.

This project proposes a sliding mode control technique for the quadrotor UAV.

The main motivation which leads us to choose this kind of control scheme, is the uncertainties of physical parameters present in this vehicle with a dynamic driving very variable as the drone. In order to deal with the uncertainties and the natural instability of the system, we are going to implement the sliding mode control technique which is known for its robustness.

The control technique consists on switching on the different sides of a decision boundary, which is called the sliding surface and the goal of the design is to enforce the error vector toward this surface during the reaching phase. Once the error vector is confined to the sliding surface, it obeys the behavior imposed by the set of equations describing the sliding surface, for example, sliding mode starts and the error vector converges to origin. [1].

Sliding mode control technique has been implemented in several systems; now let us have a look how this technique works on a draganflyer quadrotor.

2. DYNAMIC MODEL

The scheme of the quadrotor studied here is shown in Fig. 1. The equations of motion are given in [2].

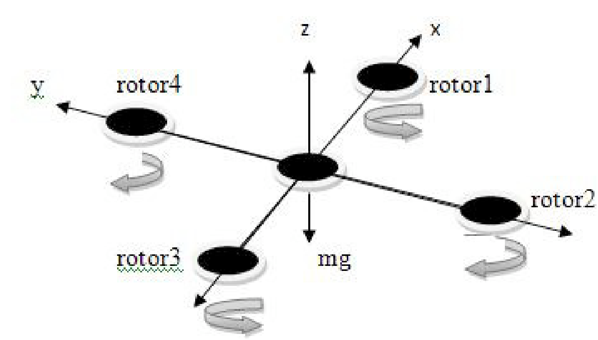

Figure 1.Scheme of the quadrotor

The quad rotor has twelve states, which are the following:

$$X = [x\ \dot{x}\ y\ \dot{y}\ z\ \dot{z}\ \emptyset\ \dot{\emptyset}\ \theta\ \dot{\theta}\ \ \psi\ \dot{\psi}]$$

here, x, y and z are the position in the x, y, and z. $\dot{x}, \dot{y}, and\ \dot{z}$ are the speed in the axes. $\emptyset, \theta, \psi$ are the pitch, roll, and yaw angles respectively, and the parameters $\dot{\emptyset}, \dot{\theta}, \dot{\psi}$ are the speed for pitch, roll and yaw.

The input signal U_1 is the total drag of the rotors. $U_2, U_3, and\ U_4$ are the moments for pitch, roll and yaw respectively. Equations (1) to (6) describe the dynamics of the quadrotor which were taken from [1].

$$\ddot{x} = \frac{\cos(\emptyset)\sin(\theta)\cos(\psi) + \sin(\emptyset)\sin(\psi)}{m}U_1 \quad (1)$$

$$\ddot{y} = \frac{\cos(\emptyset)\sin(\theta)\sin(\psi) - \sin(\emptyset)\cos(\psi)}{m}U_1 \quad (2)$$

$$\ddot{z} = -g + \frac{\cos(\emptyset)\cos(\psi)}{m}U_1 \quad (3)$$

$$\ddot{\emptyset} = \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\Omega + \frac{1}{I_x}U_2 \quad (4)$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\theta}\Omega + \frac{1}{I_y}U_3 \quad (5)$$

$$\ddot{\psi} = \dot{\theta}\dot{\phi}\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}U_4 \quad (6)$$

Where m represents the mass of the quad rotor, J_r is the inertia of the rotor and I_x, I_y and I_z are the inertia of the quad rotor in 'x', 'y' and 'z' respectively.

The input signals are described from the equation (7) to (10).

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (7)$$

$$U_2 = lb(-\Omega_2^2 + \Omega_4^2) \quad (8)$$

$$U_3 = lb(-\Omega_1^2 + \Omega_3^2) \quad (9)$$

$$U_4 = b(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \quad (10)$$

The angular speed for each rotor is noted as $\Omega_1, \Omega_2, \Omega_3, \Omega_4$

3. SLIDING MODE CONTROL

The system is going to be controlled using four control equations. The signal U1 is going to be used to define the reference of the altitude while the signals U2, U3 and U4 give references control the roll, pitch and yaw of the system. Looking at the equations (1-6), we found four control input signals. The action of these input signals makes the quadrotor moves forwards, backward, to the left, to the right, upwards or down.

The main idea to control the altitude by using sliding mode control is to make the states of the system converge to a sliding surface and make them stay on it, as it is shown in the Fig. 2. In this way, the dynamics of the system is defined by the equations that determine this surface.
Establishing these equations and making them act on the system makes possible to obtain the stabilization of the system, a precise set point following and the regulation of the variables.

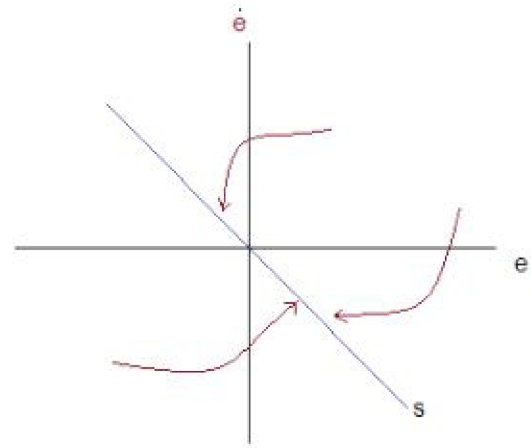

Figure 2.Sliding surface

The sliding surface, noted as "s" is defined by the equation (11)

$$s = \dot{e} + \lambda e \quad (11)$$

Where λ is a parameter design greater than zero.
In order to reach the main idea of the sliding mode control, the control law will be defined by the equation (12)

u = ueq + ucr. (12)

The equation (12) shows that the control law has two elements which are ueq and ucr. ueq is the equivalent control and this is the part of the controller that maintains the state of the system restricted to the sliding surface. ucr is the correction control and this is the part that make the state to converge the sliding surface and satisfy the following inequality:

$$s.\dot{s} < 0$$

"Geometrically, this inequality means that the time derivatives of the state error vector always point toward the sliding surface when system is in reaching mode, and therefore, the system dynamics will approach to the surface dynamics in a finite time." [4]

4. SLIDING MODE CONTROLLER

The controller is going to have the structure of the equation (12), where ueq is the equivalent control, which makes the state stays in the sliding surface s = 0 and ucr that is the correction control, which makes the state converge to s = 0.
In order to find ueq the following procedure was followed.
The sliding surface is defined as in equation (11), where "e" is the error defined as the difference between the measured state

and the desired state. For the altitude of the quadrotor the error would be.

$$e = z - z_d \qquad (13)$$

z will be the measured state and zd is the desired state.
By replacing (13) in (11) the following result is obtained.

$$s = (\dot{z} - \dot{zd}) + \lambda(z - zd) \qquad (14)$$

The law for the attractive surface is the derivative of (11).

$$\dot{s} = \ddot{e} + \lambda\dot{e}$$

$$\dot{s} = (\ddot{z} - \ddot{zd}) + \lambda(\dot{z} - \dot{zd})$$

By using the equation (3) we have:

$$\dot{s} = (-g + \frac{\cos(\emptyset)\cos(\psi)}{m}U_1 - \ddot{zd}) + \lambda(\dot{z} - \dot{zd})$$

Considering U1 = ueq + ucr the equation above becomes.

$$\dot{s} = (-g + \frac{\cos(\emptyset)\cos(\psi)}{m}(ueq + ucr) - \ddot{zd}) + \lambda(\dot{z} - \dot{zd})$$

The aim is to find ueq. In order to do this it is possible to assume that the state is on the sliding surface, so ucr and $\dot{s}$ are 0; therefore, ueq becomes:

$$u_{eq} = [g - \lambda(\dot{z} - \dot{z}_d) + \ddot{z}_d]\frac{m}{\cos(\phi)\cos(\psi)} \qquad (15)$$

In order to find ucr, a Lyapunov function V is defined. This function must be positive-definite.

$$V = \frac{1}{2}s^2 > 0$$

The derivative of the function V must be negative-definite.

$$\dot{V} = s\dot{s} < 0$$

In order to make sure that the derivative of V is negative-definite, ucr should be:

$$ucr = -sign(s).\eta \qquad (16)$$

Where s is defined by (11), **λ** and η are design parameters which are greater than zero. In this way, adding (15) and (16) the controller for the altitude is obtained.

The implementation of the controller for the altitude gives the result showed in the Fig. 3. It is seen that the output signal oscillates. This oscillation is called chattering, it can causes low control accuracy, high wear of moving mechanical parts, and high heat losses in power circuits.[8]

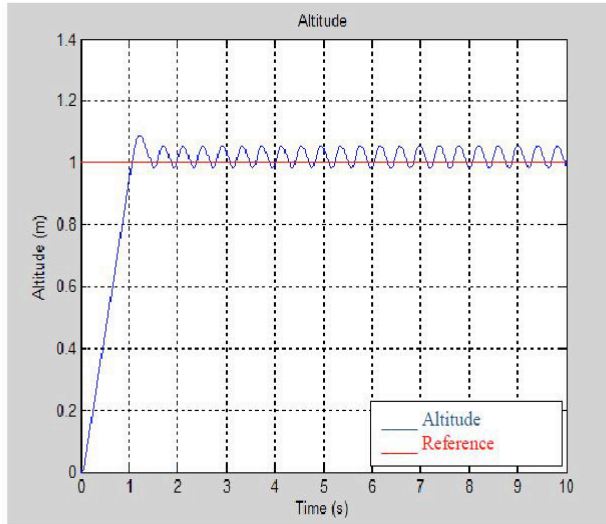

Figure 3. Altitude signal with chattering

In order to avoid chattering, saturation is added to the correction control, so ucr would be.

$$ucr = -sat(s).\eta$$

By applying this, the chattering is reduced as it is shown in the Fig. 4.

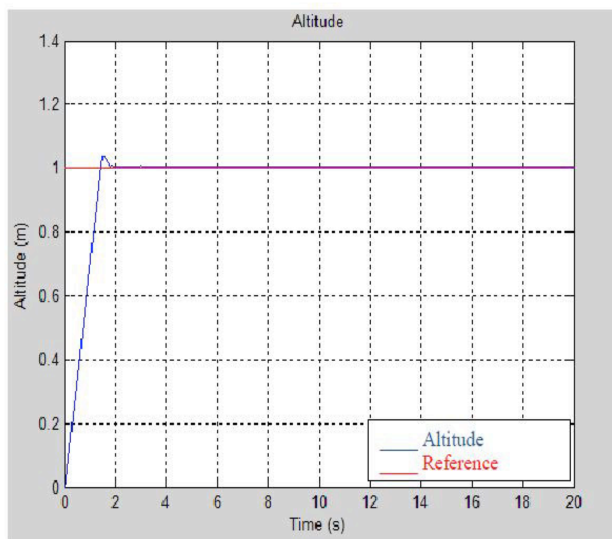

Figure 4. Altitude signal obtained by avoiding chattering using saturation

The same procedure is followed to get the controllers for pitch, roll and yaw.

$$u_{eq\phi} = \left[-\lambda(\dot{\phi} - \dot{\phi}_d) + \ddot{\phi}_d - \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right)\right]I_x$$

$$u_{eq\theta} = \left[-\lambda(\dot{\theta} - \dot{\theta}_d) + \ddot{\theta}_d - \dot{\Phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right)\right]I_y$$

$$u_{eq\psi} = \left[-\lambda(\dot{\psi} - \dot{\psi}_d) + \ddot{\psi}_d - \dot{\theta}\dot{\Phi}\left(\frac{I_x - I_y}{I_z}\right)\right] I_z$$

The $\boldsymbol{\lambda}$ and η values are detailed in the table 1, and they were found by test and error.

Table 1.Values for the controllers

	Altitude	Roll	Pitch	Yaw
λ	68	5	5	5
η	25	100	100	100

The system completely controlled is presented in the Fig.5. The block as "Altitude" is the controller for the altitude, designed through the sliding mode control method. Looking at the block the input number three, is the set point. Here it is possible to set the altitude for the quadrotor. The output of this block gives the control signal ControlZ as it is shown in the Fig. 5. Signal ControlZ gives the drag needed to lift the quad rotor, this drag is directly related to the speed of reference for the motors according to the equations (7-10); Therefore, this value of the drag will be input signal for the first stage of the block motors and the output of this stage will give the desired speed of the motors which is the reference for the speed control loop of the motors. The motors receive this referent value of the speed and automatically the needed voltage will be applied to the motors. In this way, the motors will rotate at the speed needed, giving the enough force to lift the quadrotor at the desired altitude. The signals ControlPhi, Control Theta and ControlPsi will give the moments needed to make the quadrotor rotate over x, y and z respectively. It means that ControlPhi sends the control signal to make the quadrotor perform the roll movement, ControlTheta controls the pitch movement and ControlPsi controls the yaw movement. Due to these rotational movements, the speed of the motors will change, but the system will always have the speed to maintain the quad rotor at the same altitude all the time.

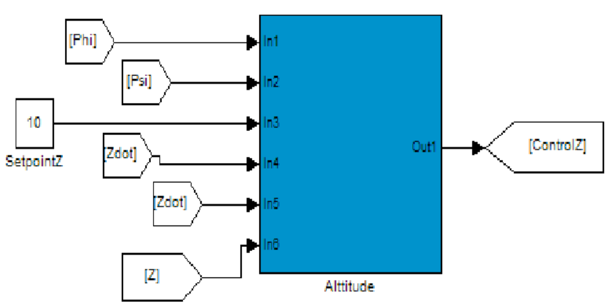

Figure 5. Control for altitude

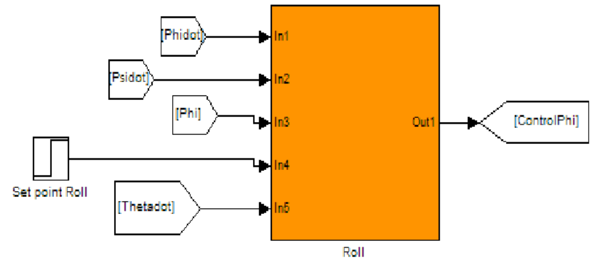

Figure 6. Control for roll

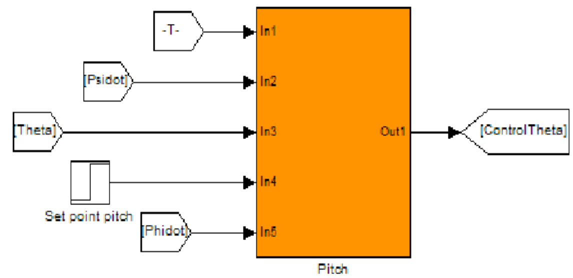

Figure 7. Control for pitch

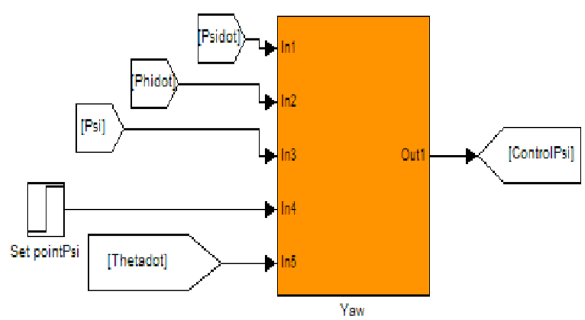

Figure 8. Control for yaw

Figures 5 to 8 are the controllers composed of (15) and (16) in order to get (12).

5. CONTROL OF THE MOVEMENTS

Fig. 9 presents the altitude of the quad rotor. In this case the altitude set point is 1m while the set point for the angles is zero radians. According to the graph the qua rotor reaches 1m at 1.5 seconds, in the beginning, it has a small overshoot, finding a maximum peak of around 1.06m before staying stable. The quad rotor is lifting smoothly during 1.5 seconds until it reaches the desired altitude. In this case, the altitude set point is 1m, the system reaches 1.006m in steady stable, which is considered as an acceptable value due to the position error is 0.6%. The altitude signal does not present chattering.

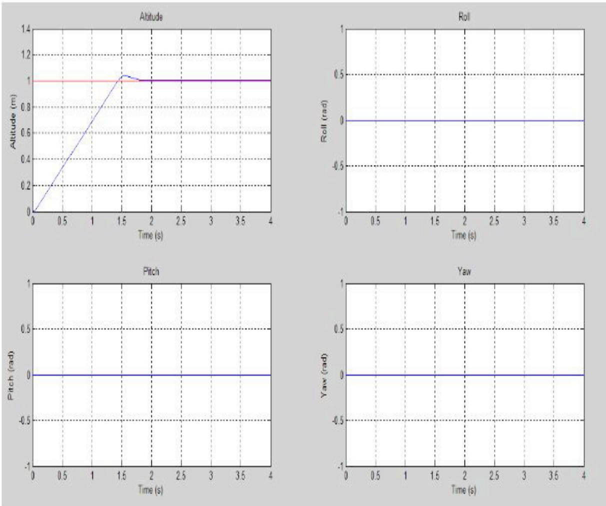

Figure 9. Altitude control by using sliding mode controller. Altitude over 1m above ground. Roll, pitch and yaw angles are set to zero.

Fig. 10 describes the behavior of the system when after 3 seconds of hovering the roll angle Phi reaches 0.1 radians. At this point, the pitch angle presents distortions, but the controller tries to keep it at zero. The yaw angle at 3 seconds has a negative peak which value is -9x10-6 radians which is a very small value that does not affect the altitude. Even though the pitch and yaw angles have distortions at 3 seconds, the altitude is stable at 1.006m, the altitude does not experience distortions at 3 seconds.

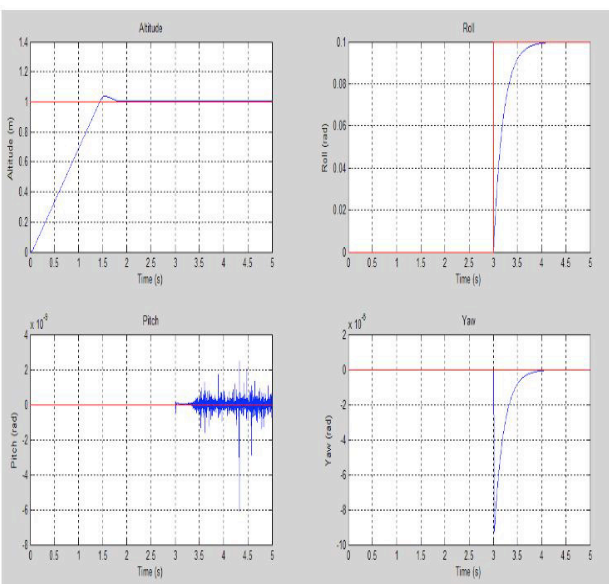

Figure. 10 Altitude control by using sliding mode controller. Altitude over 1m above ground when the roll angle is set to 0.1 rad, pitch and yaw angles are set to zero.

Fig. 10 describes the behaviour of the system when after 3 seconds of hovering the pitch angle Theta reaches 0.1 radians. At this point, the roll angle presents a neglectable positive peak of 0.5x10-10 radians during 0.5 seconds, until the pitch angle stays stable near 0.1 radians, after that the angle stays in zero. The yaw angle at 3 seconds has a positive peak which value is approximately 9x10-6 radians which is a very small value that does not affect the altitude. Even though the pitch and yaw angles have distortions at 3 seconds, the altitude is stable at 1.006m, the altitude does not experience distortions at 3 seconds.

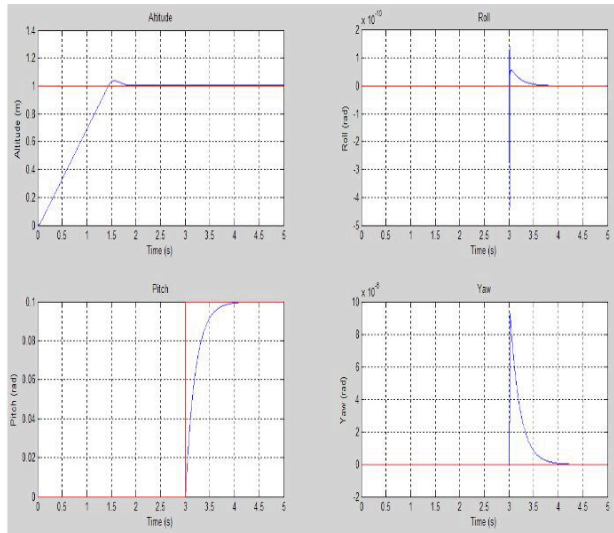

Figure 11. Altitude control by using sliding mode controller. Altitude over 1m above ground when the pitch angle is set to 0.1 rad. Roll and yaw angles are set to zero.

The Fig. 12 describes the behavior of the system when after 3 seconds of hovering the yaw angle Psi reaches 0.1 radians. At this point, the roll and pitch angles stay at zero without distortions and the altitude is stable at 1.006m. The altitude does not experience distortions at 3 seconds.

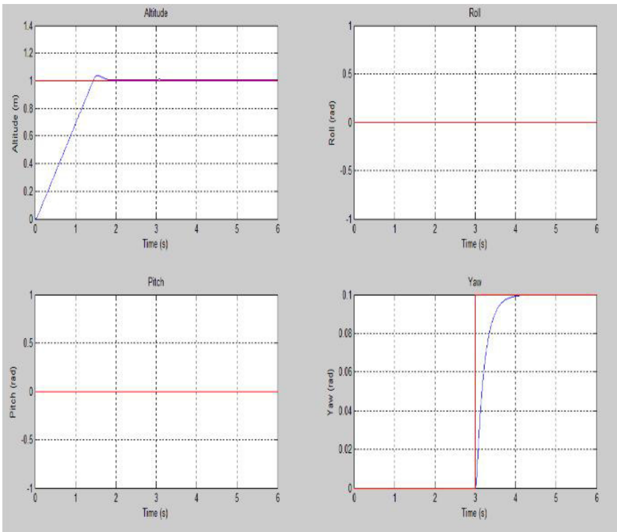

Figure 12. Altitude control by using sliding mode controller. Altitude over 1m above ground when the yaw angle is set to 0.1 rad. Roll and pitch angles are set to zero.

6. CONCLUSIONS

We have used sliding mode control to have control on the movements of a quadrotor draganflyer. Looking at the results, it is possible to say that the altitude is well controlled. It is not affected even if the quadrotor makes another maneuver such as, a pitch, yaw or roll movement. Through the sliding mode control technique we see that the system is fast and totally stable since it reaches the references set for every maneuver almost immediately and follow them in a precise manner along time.

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